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# Risk sharing markets and hedging a loan portfolio: a note[1]

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**Abstract**: Our study features a financial institute facing credit risk. Hedging credit risk by offsetting an open position with an opposite one in the financial market is important for financial intermediaries, which are concerned with both the profitability and risk of their operations. As risk management is crucial for the financial institute, the issues of how it is optimally determined and how it adjusts to changes in the financial environment deserve closer scrutiny. We extend the analysis of hedging with financial instruments against credit risk to the case of multiple types of credit risk. We show that standard results on the optimal hedge ratio and risk management effectiveness in the case of one single source of credit risk to carry over a loan portfolio in a non-trivial but intuitive way. While we focus on credit risk and credit derivatives, our analysis can be easily applied to other financial assets, which can be traded in futures market.

**Keywords**: risk management, credit risk, loan portfolio, derivatives, hedging effectiveness.

**JEL codes**: D81; G10; G21.

## Introduction

Arising from self-insurance and self-protection effects of asset-liability choices, financial firms have access to a large number of risk sharing markets which enable risk trading and improve risk management (Chen & Lin, 2016; Li & Lin, 2016). This study examines the optimal hedging decisions of a risk-averse financial intermediary (FI) facing two types of credit risk. The correlation between credit risk and the financial hedge instrument is pivotal in determining the optimal risk management. The results have implications for decision making.

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In an important contribution to the literature on risk sharing markets. Benninga et al. (1983) addressed the issue of optimal hedging in the presence of unbiased futures prices. They derived conditions for the optimal hedge to be a fixed proportion of the exposure position, regardless of the agent's risk preferences, i.e., its utility function. This result is important because of the sizeable research on theoretical and empirical hedging that abstracts from the particular utility function of risk-averse expected utility maximizers (see i.e. Wong, 1997; Broll & Wong, 2010; Freixas & Rochet, 2008).

The model used model features a risk-averse bank management facing multiple sources of risk, i.e., there are different types of credit risk. It is shown that the effectiveness of financial instruments to hedge against credit risk crucially depends on the correlation between the types of credit risks. Thus this correlation is an important factor shaping the banking firm's optimal hedging strategy.

Volatilities on global financial markets have led to the development of various futures markets. These risk-sharing markets have experienced a remarkable rate of growth throughout the world and resulted in the creation of many new financial instruments for hedging. Such hedging instruments allow a better control of risk exposure faced by national and international banking firms.

The industrial economics approach to the microeconomics of banking and finance has been supplemented with aspects of uncertainty and risk aversion.[5] This can be used to e.g. analyze credit risk, interest rate risk and political risk. Against the background of an increased importance of markets for credit derivatives various authors examined the impact of instruments to hedge against credit risk in such a framework. A typical study in this area of literature addresses the question of the optimal hedge volume and hedge ratio. If the financial instrument is perfectly correlated with the credit risk and its price is fair, a full hedge, i.e., a forward sale of all credit risk, is optimal. In the presence of basis risk, i.e., with no perfect correlation between credit risk and credit derivative, a beta-hedge rule performs best. The extent of hedging then depends on the slope of the regression of credit risk on the hedge instrument.

The study examines the case of a portfolio consisting of two different credit risks to be hedged with only one credit derivative. This is considered a suitable stylized representation of all real-world situations where there are more types of risk than financial instruments to hedge against risk. Given this modification address the optimal hedge volume is addressed and the optimal hedge ratio of a banking firm in the presence of basis risk. It turns out that the results from the standard case with only one risk and one hedge instrument carry over to this more realistic situation in a non-trivial, but still intuitive way. For example, the optimal hedge ratio is a weighted average of the exposure to the two types of credit risk.

Markets for credit derivatives have grown considerably since the nineties. From market data it is known that banks are major players in these financial

[5] See Wong, 1997; Freixas & Rochet, 2008; Broll et al., 2015, to name just a few.

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markets both as sellers and buyers of credit risk (see i.e. Minton et al., 2009). Ths approach provides one possible explanation of why a bank may want to use short and long hedging positions simultaneously. In order to focus on the banking firm's hedging motive only and ignore a motive of pure speculation, it is assumed that the hedging instrument available is fairly priced.

The plan of the paper is as follows. The next section delineates a model of a competitive banking firm in which credit risk is defined as default on interest payments. Risk sharing markets do not provide the appropriate number of perfect hedge instruments, i.e., there will be basis risk and a lack of financial instruments. The main results of this model are derived and discussed. The final section offers some concluding remarks.

## 1. The model

We develop A simple model of a banking firm that makes its hedging decisions in a one-period framework was developed . Consider a bank using given financial resources to issue loans $L$. There are two types of loans: a type 1, $L_1$, with an interest rate $r_1$ and a probability of default $\tilde{\theta}_1$, and a type 2, $L_2$ with interest rate $r_2$ and a probability of default $\tilde{\theta}_2$. The total loan volume of the bank, $L = L_1 + L_2$, is fixed. Shares $L_1/L = \alpha$ and $L_2/L = 1 - \alpha$ of the loan volume are invested in borrowers of type 1 and 2, respectively. These are thought of as having been determined before a hedging decision is taken. Note that this sequence of decisions focuses on hedging an existing loan portfolio and implies that this analysis is independent of a specific market structure in the banking industry.

There is a risk sharing market where the bank can exchange risk, denoted by the random variable $\tilde{x}$, by buying or selling a financial contract with underlying $\tilde{x}$ against a fixed payment $p_0$, the forward rate. The financial asset traded in this market can be considered as a credit derivative being more or less perfectly correlated with the two types of loans in the bank's portfolio. However this analysis is not limited to credit derivatives. Any tradeable financial asset will do. The effectiveness of the financial instrument to hedge against credit risk crucially depends on the correlations between $\tilde{x}$ and $\tilde{\theta}_i$, $i = 1, 2$. To focus on the banking firm's hedging motive as opposed to a speculative motive, it is assumed that the derivative is unbiased, i.e., $p_0 = \mathrm{E}(\tilde{x})$, where $\mathrm{E}(\cdot)$ is the expectation operator.

In empirical banking and risk management literature the usual assumption (Benninga et al., 1983) about the regressability of the random variables involved would be that

$$\tilde{\theta}_1 = a_1 + \beta_1 \tilde{x} + \tilde{\varepsilon}_1$$
$$\tilde{\theta}_2 = a_2 + \beta_2 \tilde{x} + \tilde{\varepsilon}_2$$

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where $a_i$ and $\beta_i$, $i = 1, 2$ are constants, and $\tilde{\varepsilon}_i$'s are zero–mean random variables independent of $\tilde{x}$. $\beta_i$ can take positive, zero or negative values. This is referred to as the regression dependence between $\tilde{\theta}_i$ and $\tilde{x}$. Regression dependence is formulated as a system of two seemingly unrelated regressions (SUR). Since in this case regressors are identical, in the empirical estimation the GLS estimator required for a SUR system is equivalent to equation by equation OLS (Greene 2012, p. 335). The interdependence between two types of credit risk and the price of the financial hedging instrument can be used to derive the optimum hedging policy. The econometric implementation requires assumption in addition to $E(\tilde{\varepsilon}_i) = 0$, which will be used in our analysis: Besides homoscedasticy within each regression equation and disturbances uncorrelated across observations within equations but potentially correlated across equations, strict exogeneity of the regressor is assumed. This implies that when the SUR system is estimated, $\text{cov}(\tilde{\varepsilon}_i, \tilde{x}) = E(\tilde{\varepsilon}_i \tilde{x}) = 0$. Using the SUR system with an identical regressor and implementing for an estimation of the parameters $\beta_1$ and $\beta_2$ implies that the covariance between the two credit risks to be is assumed $\text{cov}(\tilde{\theta}_1, \tilde{\theta}_2) =$ $= \beta_1 \beta_2 \text{var}(\tilde{x}) = \text{cov}(\tilde{\theta}_1, \tilde{\theta}_2)$.

The model includes two sources of basis risk. The possibility that $\text{var}(\tilde{\varepsilon}_i) > 0$ which implies that $\tilde{\theta}_i$ and the hedge instrument $\tilde{x}$ are not perfectly (positively or negatively) correlated; and the fact that there are two types of credit risks but only one hedge instrument which creates basis risk, if the credit risks are not perfectly correlated.

The banking firm is risk-averse with a von Neumann-Morgenstern utility function, $u(\pi)$, defined over its end-of-period income, $\pi$, where $u' > 0$ and $u'' < 0$. By selling (or buying) a futures contract volume $h$ based on the underlying $\tilde{x}$ the bank makes a deterministic payment $p_0$ in exchange for a stochastic claim $\tilde{x}$ per unit of $h$ at the end of the period. This stochastic claim partially or fully offsets losses in the risky loan portfolio to an extent which is controlled by the decision variable $h$ and the correlation structure of the three random variables $\tilde{\theta}_1$, $\tilde{\theta}_2$, and $\tilde{x}$. The hedge operation described then contributes $h(\tilde{x} - p_0)$ to the bank's profit.

Seen from the beginning of the period the profit function of the bank can be written as

$$\tilde{\pi} = \left[(1-\tilde{\theta}_1)\alpha r_1 + (1-\tilde{\theta}_2)(1-\alpha)r_2\right]L + h(\tilde{x} - p_0),$$

where credit default is defined as default on interest payment only and ignoring operational costs.

The bank's decision problem is then given by the expected utility maximization problem

$$\max_h E\left[u(\tilde{\pi})\right],$$

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leading to the first-order condition

$$\mathrm{E}\left[u'(\tilde{\pi}^{*})(\tilde{x}-p_0)\right]=0.$$

In the next section this condition is examined to derive properties of optimally hedging a risky loan portfolio.

## 2. Optimality of risk management

In markets for credit derivatives banks are mayor players both as sellers and buyers of credit risk (Minton et al. 2009). Our approach provides one possible explanation of why a bank may want to use simultaneously a short and long hedging positions.

### 2.1. Loan portfolio and hedging

To proceed a lemma obtained by Benninga et al. (1983) is first presented.

*Lemma:* Let $A$ and $B$ be constant and $\mathrm{E}\left[u'(A\tilde{x}+B)\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)\right]=0$. Thus we must have $A = 0$.

Assume the hedging instrument to be unbiased, i.e., $\mathrm{E}(\tilde{x})=p_0$. The main result on the optimal risk management policy of the banking firm can thus be derived.

*Proposition 1* Given a loan portfolio ($L_1$, $L_2$) and a credit derivative based on the underlying $\tilde{x}$:

1) the optimal hedge $h^*$ for the loan portfolio consisting of two types of credit risk is $h^*=h_1^*+h_2^*=\left(\alpha\beta_1 r_1+(1-\alpha)\beta_2 r_2\right)L$,

2) the optimal hedge ratio $s_i^*=h_i^*/(r_i L_i)$ for each type of credit risk $i$ is $s_i^*=\beta_i$, $i = 1, 2$,

3) the hedge ratio $s^* = h^*/(r_1 L_1 + r_2 L_2)$ for the loan portfolio as a whole is a weighted average $\beta_1\gamma_1 + \beta_2\gamma_2$ of the exposure shares of the two types of credit risk where $\gamma_1=\alpha r_1/\left(\alpha r_1+(1-\alpha)r_2\right)$ and $\gamma_2=(1-\alpha)r_2/\left(\alpha r_1+(1-\alpha)r_2\right)$.

Proof: 1. Unbiasedness of the hedge instrument implies
$\mathrm{E}\left[u'(\tilde{\pi})\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)\right]=0$, i.e.,

$$\mathrm{E}\left[u'\left\{\tilde{x}\left(\left(\alpha\beta_1 r_1+(1-\alpha)\beta_2 r_2\right)L-h^{*}\right)+\alpha\tilde{\varepsilon}_1 r_1 L+(1-\alpha)\tilde{\varepsilon}_2 r_2 L\right\}\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)\right]=0.$$

Denote $A=\left(\alpha\beta_1 r_1+(1-\alpha)\beta_2 r_2\right)L-h^{*}$ and $B(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)=\alpha\tilde{\varepsilon}_1 r_1 L+(1-\alpha)\tilde{\varepsilon}_2 r_2 L$. The above equation can be rewritten as

$$\mathrm{E}\left[u'\left\{\tilde{x}A+B(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)\right\}\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)\right]=0.$$

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Now the conditional expectation is applied and the result below is obtained

$$\mathrm{E}\left[u'\left\{\tilde{x}A + B(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)\right\}\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)\right] = \mathrm{E}\left(\mathrm{E}\left[u'\left\{\tilde{x}A + B(\tilde{\varepsilon}_1,\tilde{\varepsilon}_2)\right\}\left(\tilde{x}-\mathrm{E}(\tilde{x})\right)[\tilde{\varepsilon}_1,\tilde{\varepsilon}_2]\right]\right).$$

Due to the independence of $\tilde{\varepsilon}_i$ and $\tilde{x}$ and by using the Lemma, this can only be true, if $A = 0$ or equivalently $h^* = \left(\alpha\beta_1 r_1 + (1-\alpha)\beta_2 r_2\right)L$ .
2. The result immediately follows from 1.
3. Using 1. and the definitions of $\gamma_i$ yields the result.

A generalization of the claim to more than two types of credit risk in the loan portfolio is straightforward. It should be noticed that hedging decisions can be decentralized in the bank e.g. in profit centers for business loans and household loans. As long as each decision unit calculates its correct values $h_i^*$ and $s_i^*$, the optimal hedge ratio $s^*$ for the whole loan portfolio will hold. This result can be considered a generalization of the well-known beta-hedge rule as in Benninga, Eldor and Zilcha (1983) in the case of a portfolio. The optimal hedge strategy for each risk type is a beta-hedge. For the portfolio these strategies lead to an exposure weighted average of betas. Furthermore these results are equivalent to risk-minimization models.

From market data it is known that banks are major players in credit derivatives markets both as sellers and buyers of credit risk. This approach provides one possible explanation of why a bank may want to use not only long, but also short hedging positions without pursuing a speculative motive. Total hedge volume $h^*$ may even be negative, i.e., the bank provides insurance against credit risk to other market participants. Inspection of $h^*$ shows that this will happen for $\beta_1$ and $\beta_2$ less than zero and can happen, if one of $\beta_1$ or $\beta_2$ is negative and dominates the optimal hedge volume. Take as an example a situation where the derivative used for hedging purposes is highly correlated with the business cycle and the bank under consideration has a loan portfolio heavily biased towards borrowers from an industry where business conditions move counter-cyclically.

Adjustments of the optimal hedge volume $h^*$ to changes in parameters like beta values $\beta_i$, interest rates $r_i$ and portfolio shares $\alpha$, $(1 - \alpha)$ can be easily derived from the solution presented in this proposition.

## 2.2. Hedging performance

The risk reducing quality of a financial hedging instrument can be evaluated using the concept of a hedging effectiveness (Ederington, 1979). An index of hedging effectiveness HE* ∈ [0, 1] is defined as 1 minus the ratio of the variance of profits with hedging, $\pi$, over the variance of profits without hedging, $\pi_0$, i.e.,

$$\mathrm{HE}^* = 1 - \mathrm{var}(\tilde{\pi}) / \mathrm{var}(\tilde{\pi}_0).$$

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If two perfectly correlated and unbiased financial hedging instrument were available, the index HE* would be equal to 1. The presence of basis risk, $\text{var}(\tilde{\varepsilon}_i) > 0$, for at least one risk type $i$ leads to a less than perfect hedge, i.e., there is a residual risk. The following result can be proven:

*Corollary:* For a loan portfolio with two types of credit risk the optimum hedging effectiveness, HE*, is given by

$$\text{HE}^* = \frac{(\beta_1 r_1 L_1 + \beta_2 r_2 L_2)^2 \,\text{var}(\tilde{x})}{A}.$$

Proof: It is known that $\tilde{\pi} = r_1 L_1 \tilde{\varepsilon}_1 + r_2 L_2 \tilde{\varepsilon}_2 + c_1$ and $\tilde{\pi} = (\beta_1 r_1 L_1 + \beta_2 r_2 L_2)\tilde{x} + r_1 L_1 \tilde{\varepsilon}_1 + r_2 L_2 \tilde{\varepsilon}_2 + c_2$ with $c_1$, $c_2$ being some constants and where $A = \text{var}(\tilde{\theta}_1)(r_1 L_1)^2 + \text{var}(\tilde{\theta}_2)(r_2 L_2)^2 + 2 r_1 L_1 r_2 L_2 \text{cov}(\tilde{\theta}_1, \tilde{\theta}_2)$.

In the optimum for a single risk type under the beta-hedge rule $\text{HE}_i^*$ can be shown to be equal to the square of the correlation coefficient $\rho_i$ between the future price $\tilde{x}$ to represent the future price of the derivative and the credit risk, i.e., $\text{HE}_i^* = \rho_i^2$. This can also be written as

$$\text{HE}^* = \frac{\beta_i^2 \,\text{var}(\tilde{x})}{\beta_i^2 \,\text{var}(\tilde{x}) + \text{var}(\tilde{\varepsilon}_i)}.$$

Notice the generalization here of hedging effectiveness to the case of a portfolio of loans which exhibits a structure similar to the one-risk case.

The hedging effectiveness (HE) can be used to evaluate the risk-reducing quality of alternative financial instruments for hedging purposes. In the case of a portfolio of credit risks all derivatives available will typically only provide an imperfect hedge. Therefore the bank has to choose from a set of instruments. Index HE offers an operational measure for this selection problem. As mentioned earlier, not only credit derivatives are candidates for hedging a loan portfolio. Other derivatives such as macro derivatives, can also deliver the risk reduction sought by a bank.

## Conclusions

In this study the analysis of hedging credit risk with a credit derivative was extended to the case of a loan portfolio. It was found that the optimal hedge volume is a weighted average of the exposure shares of the two types of credit risk. The hedging effectiveness of the optimal hedge turns out to be a straightforward generalization of the hedging effectiveness in the single-risk case.

It should be emphasized that these results are not limited to the use of a credit derivative. Any financial instrument could replace the credit derivative consid-

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ered in this decision making model. All that needs to be known are the slopes of the regressions of the two credit risks on the risk of the instrument. In particular a macro derivative could serve as a substitute for the credit derivative. From the literature it is known that such a macro derivative not only carries the advantage of enabling the bank to trade only the systematic part of credit risk, but also has analytic properties analogous to a credit derivative with basis risk of the type examined here. The analysis applies to all types of risky assets on a bank's balance sheet.

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